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HYBRID COMPUTER TECHNIQUES FOR DETERMINING PROBABILITY DISTRIBUTIONS

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ABSTRACT

This report describes an automated method of calculating and displaying the instantaneous amplitude probability distribution (IAPD) and the peak amplitude probability distribution (PAPD) of a random signal recorded on magnetic tape. A special digital logic computer is combined with a simple analog computer to provide the unique circuits necessary for computing these data.

A particular application is described that proved to be valuable in dynamic-stability studies of space vehicles. The accuracy and economics of the method are discussed.

INTRODUCTION

A large number of problems in engineering and science involve the concepts of probability and statistics; that is, they can be described adequately only in a statistical manner. A discussion of probability and statistics can be found in many texts.(1)

In a strictly academic problem, it is relatively easy to determine probability distributions. This is not always true in a practical problem, however, where it is often difficult to obtain such data.

In many problems, the information to be analyzed is the system response recorded on a magnetic tape. The probability curve cannot be determined mathematically since we do not have equations, and often have little knowledge of the forcing function.

It is a long, tedious, and costly job to obtain probability distributions from recorded signals if the information is analyzed graphically by hand.

DISCUSSION

Definition and Assumptions

This report describes an accurate and economical method of calculating and displaying the IAPD and the PAPD of a random signal recorded on a magnetic tape. A special digital logic computer is combined with a simple analog computer to provide the unique circuits used to compute these data.

Pressure cell and strain gage measurements were made of turbulent air flow and structural oscillations of a space-vehicle and launch-vehicle combination during ascent through the atmosphere. The purpose was to determine the suitability of different combinations of space vehicles, boosters, and escape tower shapes on the dynamic stability of the space launch system. The signals were recorded on magnetic tape using FM. The information on the tape was then fed to the computer to determine the probability distribution curves. A cycle of a typical information signal and the rectified signal are shown in figure 1.

The basic assumptions made in designing the circuit were: (1) signal samples are statistically independent, (2) the frequencies of interest are between 10 cps and 150 cps.

The following terms are defined for clarity in interpreting this report.

IAPD: This is understood here as the percentage of time a random signal is above some arbitrary

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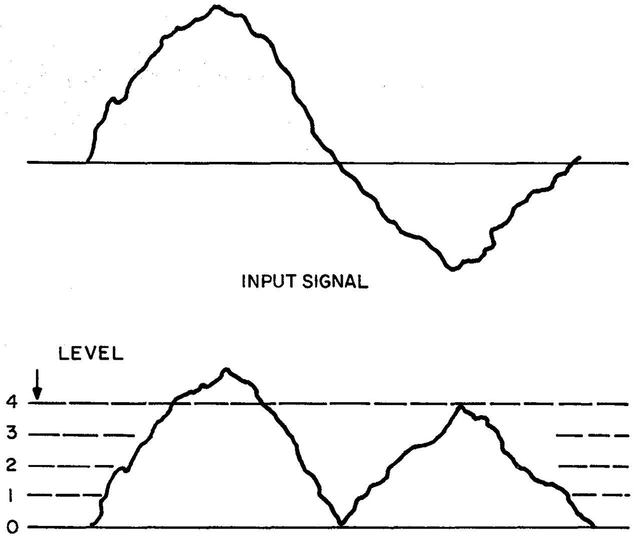


Figure 1. One Cycle of a Typical Information Signal

level, where there are n arbitrary levels to provide the distribution comparison(2).

PAPD: This is identical to the IAPD except that comparison is made for the percentage of the total number of peaks that are above some arbitrary level. This can also be visualized as the IAPD of the envelope of the signal.

Root-Mean-Square (RMS): The normal meaning is used here; that is,

$$E_{RMS} = \sqrt{\frac{1}{T} \int_{t=o}^{t=T} \left[f(t)\right]^2 \, dt}$$

where T is the signal period. In the cases studied, the period was assumed to be the time duration of that particular computer run under analysis. This interpretation is valid since the wave form can be assumed to repeat itself in the following equal time duration.

Absolute Value: This term is synonymous with rectification as used in this report; that is, all negative portions of the signal are inverted after any d.c. components have been removed (see fig. 1). This can be expressed as:

$$\left| f(t) \right| = \left| f_1(t) - \frac{1}{T} \int_o^T f_1(t) \, dt \right|$$

where $f_1(t)$ is the signal recorded on the tape. This equation was simplified for purposes of simulation by replacing $1/T \int_o^T f_1(t)\,dt$ by a low pass filter with a transfer function, $1/(1+\tau_1 s)$, where T is the time for one run, τ_1 is unity, and s is the Laplacian operator.

Counter: A counter is a circuit, constructed of flip-flops and gates, that is capable of counting either a number of events or a time duration. The counter, when properly enabled, can count either up or down and in either binary or decimal.

Digital Comparator: A comparator is a digital device capable of accepting two analog inputs and producing either a high or low output voltage (i.e., 1 or 0) dependent on the sign of the sum of the inputs. The information flow diagram and the computer logic block diagram are shown in figure 2.

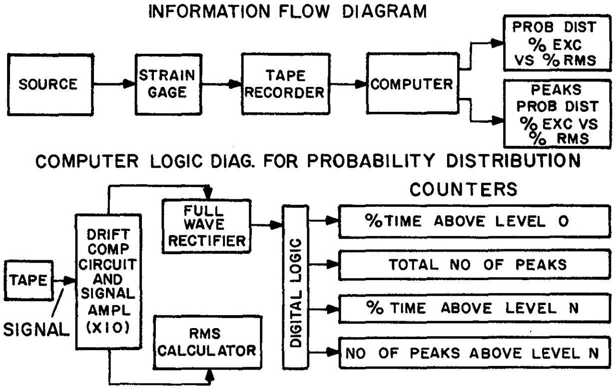


Figure 2. Block Diagram

Signal Preparation

The signal, recorded on magnetic tape, must be processed before it is analyzed in the digital logic computer. This processing must include the following: (1) signal amplification, (2) drift compensation (trend removal), (3) rectification, (4) level selection, (5) sampling rate selection, and (6) RMS calculation.

The computer circuit to accomplish items 1, 2, and 3 above is shown in figure 3. The recorded signal from the tape has a maximum value of 1.414 volts. This signal is multiplied by a factor of 10 in amplifier 00. Amplifier 01 is a low pass filter with a 1 radian/sec cutoff. This low frequency signal is inverted and subtracted from the input signal to amplifier 00. The low frequency components are assumed to be due to drift in the measuring and recording devices. Amplifiers 03 and 04 are wired to calculate the absolute value of the amplified signal, 10 f(t).

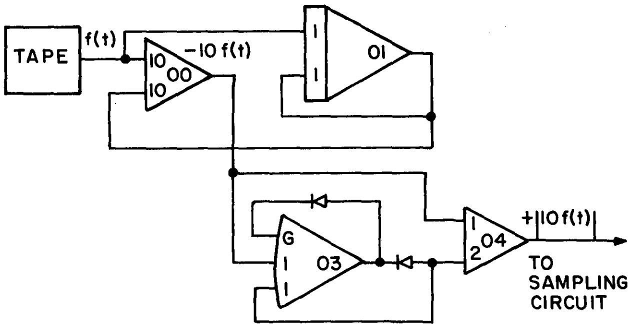


Figure 3. Analog Computer Circuit

The circuit to generate the RMS of the information signal continuously is shown in figure 4. Quarter square multipliers, M1G and M1J, were used for squaring and square rooting. A servo-multiplier, SM00, was used in the division circuit to obtain higher accuracy with low input signals.

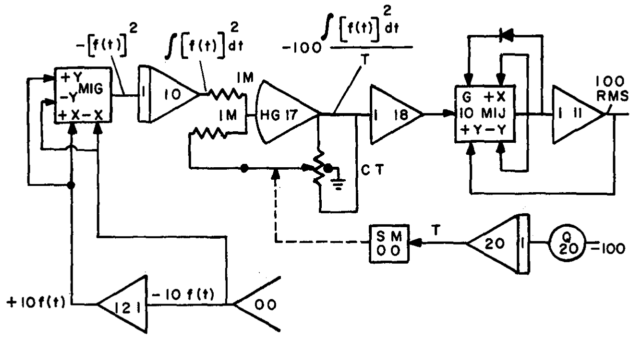


Figure 4. RMS Circuit

A typical information signal and its rectified signal are shown in figure 5. The majority frequency content is about 20 cps in this case.

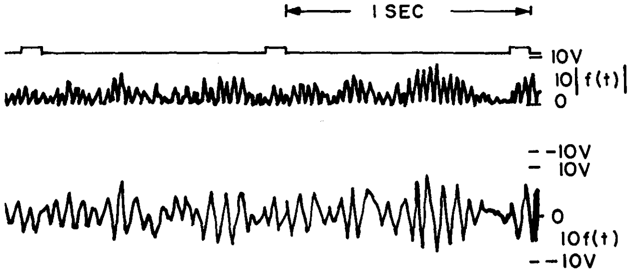


Figure 5. Typical Information Signal

Sampling Rate

The sampling rate was varied by adjusting the variable clock frequency which was set to 333 pulses per second. Using 10,000 counts as the base time rate, the computer sampled for 30 seconds, which approximated the time duration of each data record.

The sampling theorem states: If a wave form, (f)t, is band limited to frequencies between 0 and W cycles per second, then it is completely determined by its samples taken at a sampling interval, T = 1/2W seconds, i.e., a sampling rate of 2W per second.

The minimum sampling rate needed in this study was 2W = 2(150) = 300 samples per second. The rate used was about 333 samples per second. The upcounters and downcounters count one digit or bit on each clock pulse when the carry-in is high. The sampling rate can easily be adjusted for analyzing data with higher or lower frequency components.

Level Selection

The signal was quantized into eight discrete levels, counting the zero volt level. The number of levels was determined by the number of computing components available at the time. The signal sampling levels were between 0 and 1.6 volts. The incoming signal was multiplied by 10, allowing the levels to be selected between 0 and 16 volts, thus giving higher resolution. The levels selected here were 0, 4.0, 6.0, 8.0, 10.0, 12.0, 14.0, and 16.0 volts. The sampling levels could be chosen differently since there is no requirement that they be evenly spaced.

Instantaneous Amplitude Distribution

In this particular signal analysis, nine arbitrary threshold values were used for the IAPD, the lowest being zero volts and the highest being infinite volts. The signal must be above zero volts 100 percent of the time and above infinite volts 0 percent of the time. There are seven meaningful levels between these two extremes.

The method used to measure the percentage of time the signal exceeded any particular level was to assume that initially the signal exceeded that level 100 percent (or 99.99 percent) of the time, and then to subtract the amount of time the signal was below that level. This is accomplished by dividing the time base into 10,000 equal increments and subtracting one digit at the end of each time increment the signal is below that particular level.

The circuit to calculate and display the amplitude distribution is shown in figure 6 (the nomenclature is listed in figure 8).

Potentiometers 80, 81, etc., set the levels at which points will be computed for the amplitude and peak distribution curves.

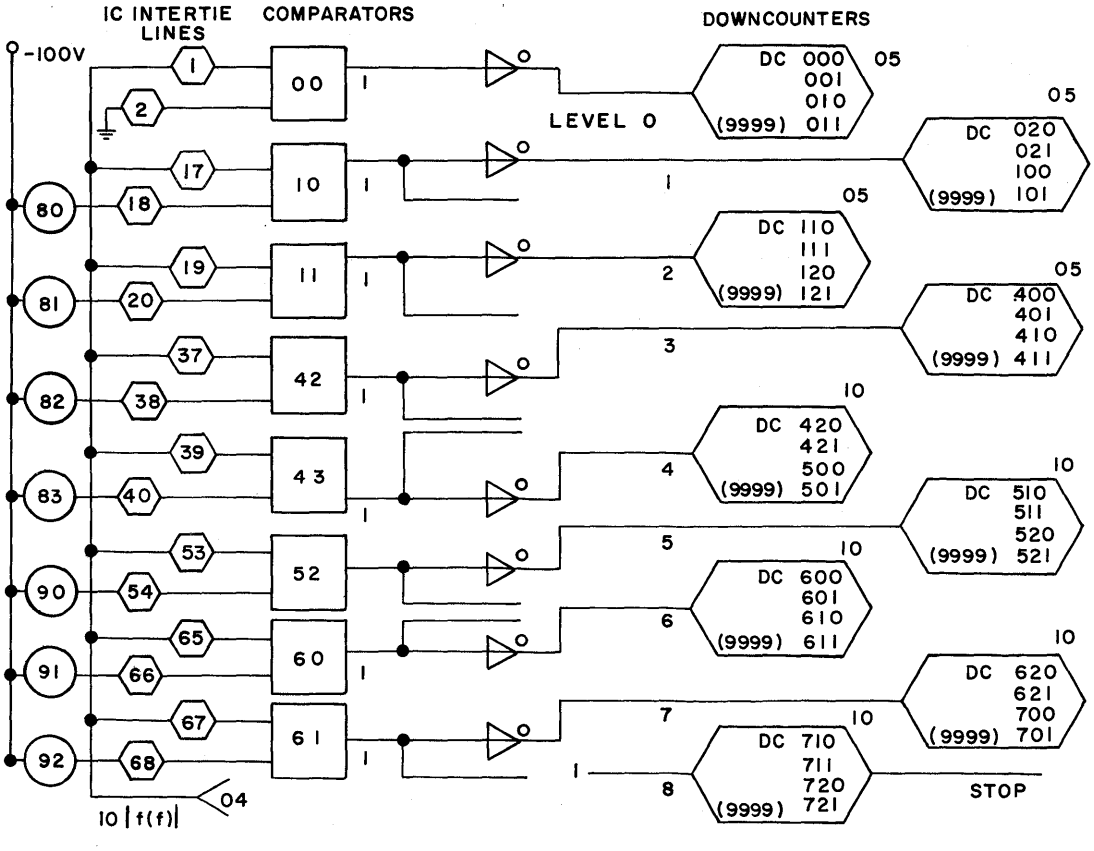


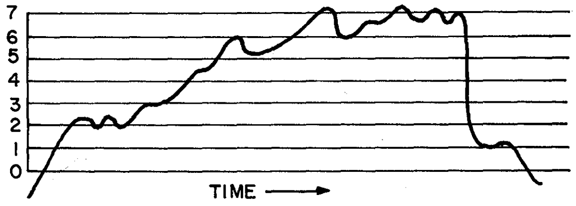
Figure 6. Instantaneous Amplitude Distribution Circuit

The IC intertie lines are trunk lines between the analog computer and the digital logic computer. In figure 6, the signal, 10 | f(t) | , is compared with the set levels from the potentiometers to operate the comparators 00, 10, 11, etc. The comparators have two outputs, 0 and 1. One output is high (1) and the other low (0) at all times. When the sum of the input voltages is positive, the 1 output is high; when the sum is negative, the 0 output is high. The switching sensitivity is 10 mv.

The 0 output of a comparator drives a downcounter that is preset to 9999. At each clock pulse that the 0 output of the comparator is high, the counter will count down one digit. The counter at level 8 has a high input at all times and therefore counts down one digit at each clock pulse. When this counter reaches 0000, it produces a carryout that stops the digital and analog portions of the computer and allows the answer to be read out.

Peak Amplitude Distribution

The arbitrary threshold values chosen here were the same as for the amplitude distribution study. This allowed the same comparators to be used. There are several definitions for amplitude peaks of a random signal. The definition to be used must be determined by the physical meaning of the peaks when interpreted in terms of the physical system.



PEAKS ABOVE

LEVEL	0	1	2	3	4	5	6	7
DEFIN 1	1	1	3	1	1	1	2	4
DEFIN 2	1	1	1	1	1	1	1	1
DEFIN 3	7	7	7	5	5	5	4	4

Figure 7. Number of Peaks vs Peak Definition

Three definitions will be discussed here, only one of which was used. Circuits will be described for all three definitions:

1. The number of peaks above a particular level can be defined as the number of times the signal crosses that level with a positive derivative.

2. A peak can be defined as the highest absolute voltage occurring during the interval between two zero crossings. A peak above level n̲ is also a peak above any lower level (i.e., n-1, n-2, etc.).

3. The number of peaks can be defined the same as in 1 above, except that if the signal crosses level n̲ with a negative derivative and then again with a positive derivative before it crosses level n-1, an additional peak is counted for level n̲ and all lower levels (i.e., n-1, n-2, etc.).

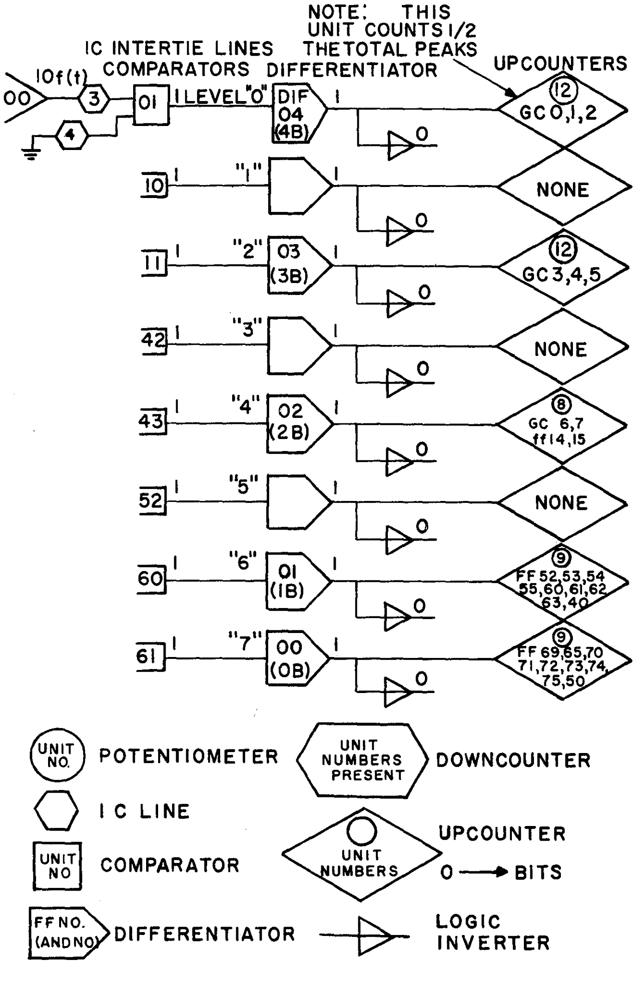


Figure 8. Peak Amplitude Distribution Circuit

Figure 7 shows how the peaks would be counted if each of the above definitions were used. The circuits for the three definitions are shown in figures 8, 9, and 10.

The number of peaks above level 0 (i.e., zero volts) is also the total number of peaks. To conserve equipment, binary upcounters were used to count the number of peaks. The amount of equipment available limited the number of levels at which peaks could be counted. Peaks were counted at only five of the eight set levels.

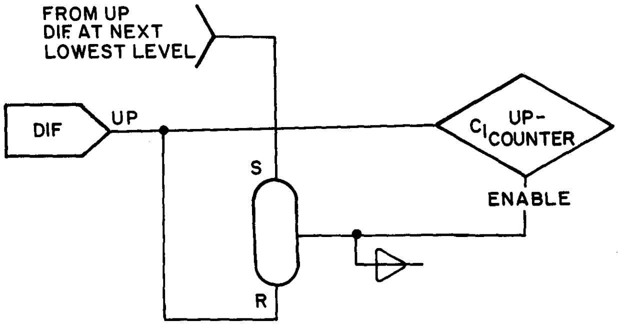


Figure 9. Peak Definition No. 2 – Additional Circuit to be added to all Levels but the Lowest (Level 0)

The amplitude peak distribution circuit, to match definition 1, is shown in figure 8. The comparators are the same ones shown in figure 6. In this case, the comparator outputs drive one - way digital differentiators.

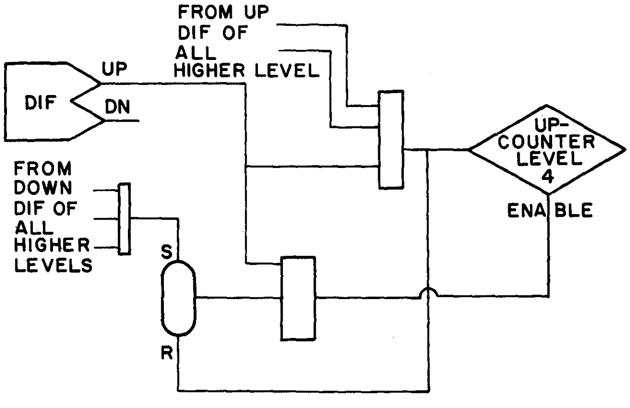


Figure 10. Peak Definition No. 3 – Additional Circuit to be added to all Levels but the Highest (Level 7)

A one-way differentiator produces a blip one clock pulse wide when the input changes from the low level to the high level (i.e., goes from 0 to 1). A circuit for this type of differentiator is shown in figure 11.

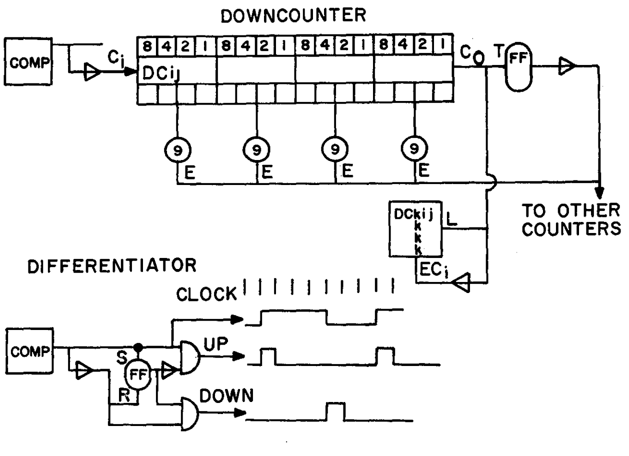
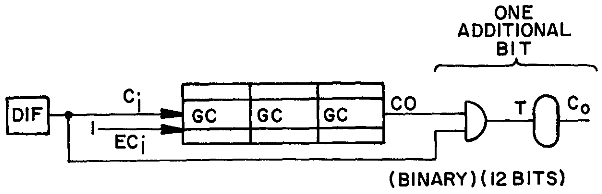


Figure 11. Digital Circuits

The output of the differentiator drives a binary upcounter. Two upcounter circuits are shown in figure 12. When the input to the counter is high, it will count up one bit at each clock pulse. Since the differentiator is high for only one clock pulse, the counter will add one bit each time the comparator flips high, that is, each time the increasing signal crosses the set level.

The signal, 10 f(t), rather than the absolute value, 10 f(t), drives the counter in level 0. This gives assurance that the signal will pass through zero. The counter in level 0 counts only half the total peaks since the negative portion of the signal has no effect on the comparator.



9 BIT UP COUNTER

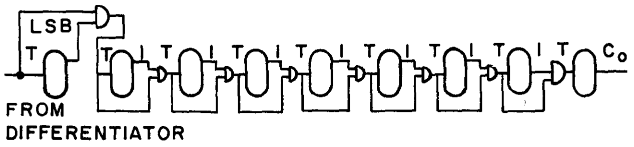


Figure 12. Digital Circuits

The number of peaks at each level are read out in binary, but if a proper data sheet is used, they are easily converted to decimal values.

The amplitude peak distribution 2 is the same circuit as figure 8, with the addition of the logic shown in figure 9. The peak count at level 0 will remain the same, but the circuit will count peaks at any other level, n, only after the previous lower level, n-1, has added at least one additional peak after level n has added a peak.

The amplitude peak distribution circuit to match definition 3 is the same as figure 8 with the addition of the logic shown in figure 10. The additional circuitry will perform the following functions:

1. For positive derivative crossings, it will count a peak at that level, n, and at all lower levels, n-1, etc., if these lower levels have been enabled.
2. It will deactivate lower level counters after count.
3. For negative derivative crossings, it will enable all lower level counters and disable all higher level counters.

RESULTS

If the IAPD curve falls on the normal distribution curve, the PAPD curve should, theoretically, fall on the Rayleigh distribution curve. In these studies, the results have been normalized to the RMS value of the sample. Although the PAPD could conceivably fall below the normal distribution, it is not physically possible for the PAPD curve to fall below the IAPD curve.

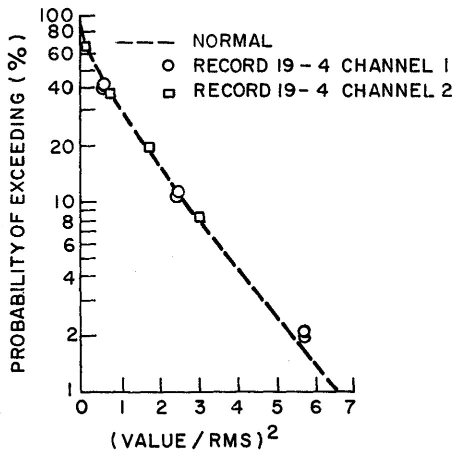


Figure 13. Probability Distributions Comparisons

The results obtained agree favorably with the normal distribution curve. A comparison is made in figure 13. It can be seen that the computer data from a typical information signal are very close to a normal distribution curve.

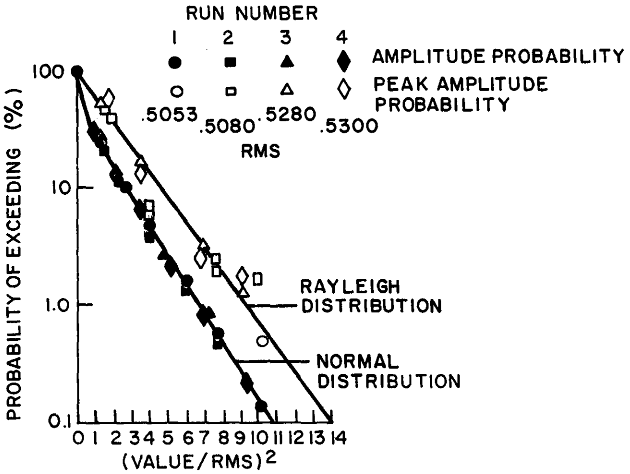


Figure 14. Probability Distribution of White Noise

Figure 14 shows the instantaneous amplitude and peak amplitude probability distribution curves of so-called white noise* calculated with the computer setups described here for peak definition 1. The instantaneous amplitude probability is very close to the theoretical curve(2). The peak amplitude probability is close to the Rayleigh curve except for one scattered point. The PAPD curve will, theoretically, have the Rayleigh distribution if the peak sampling describes the envelope of the signal. If the peaks are used to describe a new curve, this curve can be considered to be the envelope of the signal. The scattering is probably caused by the high frequencies in the noise. High frequencies might cause some inconsistencies in the results since the computing circuit was designed to handle frequencies up to 150 cps. This figure also shows the repeatability from run to run. Four sets of data are shown.

VOLTAGE	LEVEL	AMPLITUDE % ABOVE			NO OF PEAKS		
		RUN A	RUN B	RUN C	RUN A	RUN B	RUN C
0	0	99.99	99.99	99.99	1232	1214	1208
.4	1	42.35	41.92	42.16			
.6	2	22.09	22.40	22.94	575	584	590
.8	3	9.41	9.74	10.05			
1.0	4	3.13	3.29	3.38	129	121	127
1.2	5	0.94	0.95	1.02			
1.4	6	0.31	0.32	0.25	10	15	14
1.6	7	0.02	0.02	0.02	2	2	2
	8	0.00	0.00	0.00			

Figure 15. Repeatability Performance

When the same signal was analyzed on different days, the repeatability was excellent. A comparison is shown in figure 15 of the results from one signal that was analyzed at different times. The data spread is very small. The maximum deviation is at the lower levels where the count is highest. Where the total count is very small, there is practically no deviation. The repeatability is excellent for both the instantaneous amplitude and peak data.

The computer and all three of the peak definitions were used to calculate the PAPD of white noise. A comparison of the three curves is shown in figure 16. Definition 1 gave the most pessimistic result. Definitions 2 and 3 each produce a lower, more optimistic curve. Definition 1 was used in determining all of the results.

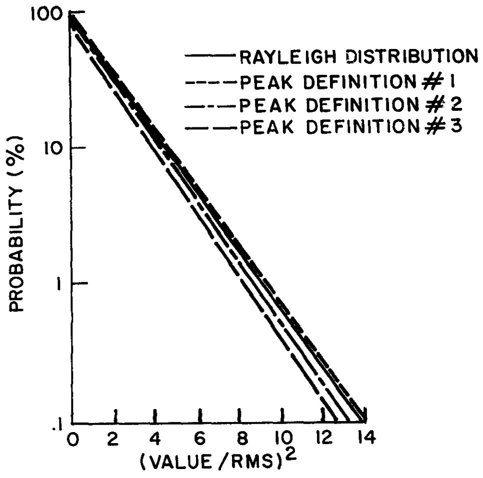


Figure 16. Peak Amplitude Probability Distribution from Three Peak Definitions

This method of determining probability curves is very economical as compared to other methods. The same data that it takes five to seven man-days to reduce by hand can be reduced in 30 seconds by the method discussed in this report. Also, the computer method is more reliable.

It is not possible at this time to compare cost figures with a general purpose digital computer method, but it is believed that the digital computer would be more costly.

The cost of using all-analog computing to analyze this data would be the same as the hybrid method. However, the repeatability and accuracy of the analog would be inferior.

The versatility of the method and equipment is emphasized by the fact that the definitions, sampling

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*The white noise used here was band limited to 35 cps and lower.

rate, and level selections can be changed and readily mechanized on the computer.

CONCLUSIONS

This paper points out a special application of two general purpose computers, analog and digital.

Advantages in using this method of data analysis are: high speed solution and confidence in results.

The analysis method described here could be applied to "on line" data analysis by adding an automatic print out and feeding the information directly to the computer.

The accuracy obtainable is limited mostly by the amount of equipment available. Higher frequency signals require a faster sampling rate and, consequently, more counting capacity.

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